

$$d_\gamma F_{\alpha\beta} + d_\alpha F_{\beta\gamma} + d_\beta F_{\gamma\alpha} = 0.$$

$$D_\gamma F_{\alpha\beta} + D_\alpha F_{\beta\gamma} + D_\beta F_{\gamma\alpha}$$

$$= F_{\alpha\beta,\gamma} - \Gamma_{\alpha\delta}^\mu F_{\mu\beta} - \Gamma_{\beta\gamma}^\mu F_{\alpha\mu}$$

$$+ F_{\beta\gamma,\alpha} - \Gamma_{\beta\alpha}^\mu F_{\mu\gamma} - \Gamma_{\alpha\gamma}^\mu F_{\beta\mu}$$

$$+ F_{\gamma\alpha,\beta} - \Gamma_{\gamma\beta}^\mu F_{\mu\alpha} - \Gamma_{\alpha\beta}^\mu F_{\gamma\mu}$$

$$= F_{\alpha\beta,\gamma} - \Gamma_{\alpha\delta}^\mu [F_{\mu\beta} + F_{\beta\mu}]$$

$$+ F_{\beta\gamma,\alpha} - \Gamma_{\beta\alpha}^\mu [F_{\mu\gamma} + F_{\gamma\mu}]$$

$$+ F_{\gamma\alpha,\beta} - \Gamma_{\alpha\beta}^\mu [F_{\mu\gamma} + F_{\gamma\mu}]$$



These terms vanish by antisymmetry

$$F_{\mu\nu} = d_\nu A_\mu - d_\mu A_\nu$$

$$= \boxed{F_{\alpha\beta,\gamma} + F_{\beta\gamma,\alpha} + F_{\gamma\alpha,\beta} = 0}$$